Mathematical Model of Optimization of Selection Distribution during Management of the "Reservoir-Well" System

Suvonov Olim Omonovich
Associate Professor of the Department of Informatics, Navoi State Pedagogical Institute

Annotation: The article discusses the modeling of the process of optimal control of a system with distributed parameters on the example of the distribution of selections in the development of oil fields to analyze the state of the system; the method of statistical tests (Monte Carlo) is used.

Keywords: mathematical model, optimal control, differential equation, boundary conditions, statistical testing method, random processes, sampling distribution, reservoir, well, indicators, oil content. If

Introduction.

To build mathematical models for analyzing the control processes of a hydrodynamic object, it is necessary to have communication operators for the field of distributed parameters. Based on the hydrodynamic regularities of the processes, these operators can be the equations of mathematical physics [1].

The "reservoir-well" ("R-W") system is the main object of oil and gas production and, as a control object, is connected with some other systems in interaction with external and internal random processes that have a certain impact on well flow rates.

The R-W system represents by number $n \geq 1$ interacting elements and the state of which is described by differential equations in partial derivatives, depending on spatial and temporal coordinates. In particular, when $n = 1$ we have a system with distributed parameters, since at each singular point of a multiply connected domain of definition there can be a concentrated source that changes in time.

If the system state parameters are stochastically distributed in the domain of definition, then the system is a system with stochastic distributed parameters. With a statistical approach, specific hydrodynamic processes occurring in an object are considered as the interaction of random processes and fields. With small perturbations that are typical for normal operation, many objects with distributed parameters can be considered as stationary in time and linear.

Many of the factors that determine the process of oil (gas) movement in the reservoir, with a limited possibility of their direct measurement, large inertia, processes occurring in the reservoir, can be assessed after a long period of time, after one or another impact on the reservoir (promotion of bottom and contour waters, various injections), changes in reservoir properties depending on the operating mode and as the energy of oil (gas) reserves is depleted.

These features of the functioning of the R-W system put forward special requirements for the methods of mathematical modeling of an object.

One of the methods of mathematical modeling is the method of statistical tests (Monte Carlo method). The Monte Carlo method has been widely used in various fields of science and technology in recent years.

1. For the first time, the Monte Carlo method is used to solve the problem of optimization of selections in the management of reservoir development. A probabilistic-theoretic walking model is presented to obtain a matrix in terms of flow rates or pressures, which is necessary to solve the problem of optimal control of the R-W system, taking into account the advancement of the dynamics of water-oil contact.
2. The implementation on a computer of simulation models of stochastic distributed objects, models of random fields, measured object (system, process) state vectors under conditions of a priori uncertainty, as well as simulation schemes of adaptive measurement algorithms (or real models) and sources of errors in the study of their properties leads to the need to use, on the one hand, stochastic models, and on the other hand, unified, efficient and optimal (in terms of computational complexity) methods and algorithms for their implementation [2 - 6].

The article will show the application of the Monte Carlo method for solving the problem of static optimization of the distribution of withdrawals from the reservoir, which is of great national economic importance, taking into account geological and technical restrictions on the parameters of the R-W systems.

Main part

Let Ω- a bounded region of two-dimensional space, the boundary of which Γ consists of an outer contour Γ_0 and internal contours $\Gamma_1, \Gamma_2, ..., \Gamma_n$ (contours of production wells). The state of the system (pressure distribution in the reservoir). The state is determined by a linear partial differential equation of elliptic type

$$L(P) = -\sum_{i=1}^{2} \frac{\partial}{\partial x_i}(K(x_i)\frac{\partial P}{\partial x_i}) = 0, \qquad x \in \Omega \qquad (1)$$

under boundary conditions

$$P(x_1, x_2) \Big/_{\Gamma} = 0 \qquad (2)$$

$$P(x_1, x_2) \Big/_{\Gamma_\nu} = \begin{cases} 0, & \nu = 1,2,...,n-1, n+1,...,m \\ 1, & \nu = n \end{cases} \qquad (3)$$

the solution of which makes it possible to obtain n- th row of the matrix of coefficients of influence and mutual influence along the flow. There $K(x_1, x_2)$- multiply connected area parameter, $P = P(x_1, x_2)$ - potential at a point with coordinates $(x_1, x_2) \in \Omega$.

When conditions (1) - (3) are met, it is required to determine such values of the control parameter (well flow rates) that the quality criterion

$$Z(q) = \sum_{i=1}^{n} c_i q_i \qquad (4)$$

(the amount of recoverable clean oil) took the maximum value

$$Aq \le B, \ (5)$$

$$q \ge 0 \ (6)$$

Single potential jump on n- m inner loop Γ_n leads to an increase in flows $Q_{n1}, Q_{n2},..., Q_{nn}$, which can be determined by the formula [2]

$$Q_{nr} = -\iint_{\Gamma_\nu} K(x_1, x_2) \frac{\partial P}{\partial n} \alpha \Gamma_\nu .$$

These functions are called influence coefficients $(n = \nu)$ and mutual influence $(n \neq \nu)$ by flow (debit)

Solving a problem of the form (1) - (3) with $n = \overline{1, m}$, we get a square symmetric matrix

$$A_Q = \begin{pmatrix} Q_{11} & Q_{12} & \cdots & Q_{1n} \\ Q_{21} & Q_{22} & \cdots & Q_{2n} \\ . & . & \cdots & . \\ Q_{n1} & Q_{n2} & \cdots & Q_{nn} \end{pmatrix}$$

To obtain a matrix of coefficients of influence and mutual influence on the potential, it is necessary to solve m boundary value problems described by equations (1), (3) and (7):

$$q_r = -\iint_{\Gamma_\nu} K(x_1, x_2) \frac{\partial P}{\partial n} d\Gamma_\nu = \begin{cases} 0, \nu = 1, 2, \ldots, n-1, n+1, \ldots, m \\ 1, \nu = n \end{cases} \quad (7)$$

Then Γ_ν - radius circle R_ν, and the fluid inflow to the well is assumed to be radial, then condition (7) can be reduced to the form

$$\frac{\partial P}{\partial n} /_{\Gamma_\nu} = -\frac{q_\nu}{2\pi K R_\nu}.$$

In particular, when $q_\nu = 1$

$$\frac{\partial P}{\partial n} /_{\Gamma_\nu} = -\frac{1}{2\pi K R_\nu}.$$

this equation is a boundary condition of the second kind.

The solution of problem (1), (2) and (7) exactly coincides with the corresponding elements n - th row of the matrix of coefficients of influence and mutual influence by potential

$$A_p = \begin{pmatrix} P_{11} & P_{12} & \ldots & P_{1m} \\ P_{21} & P_{22} & \ldots & P_{2m} \\ . & . & \ldots & . \\ P_{n1} & P_{n2} & \ldots & P_{nm} \end{pmatrix}. \quad (8)$$

Let us consider a method for determining the coefficients of influence and mutual influence, based on the application of the Monte Carlo method [7,8].

Let us set on all internal contours of a multiply connected region a single boundary condition of the first kind

$$P(x_1, x_2)\big/_{\Gamma_\nu} = 1, \qquad \nu = \overline{1, n}. \qquad (9)$$

When determining the solution to the boundary value problem (1), (2), (9) by the Monte Carlo method at the node (i, j) at each trial, a random variable ξ will take on a value equal to

$$T_{ij}^{q_0} = \begin{cases} 0, \text{ if } & (i,j)^{q_0} \in \Gamma, \\ 1, \text{ if } & (i,j)^{q_0} \in \Gamma_\nu, \nu = \overline{1, m}. \end{cases}$$

Node Potential $(i, j)^0$ will be equal to the mathematical expectation of this random variable:

$$M_\xi = Y_{ij}^0 = \sum_{q_0=1}^{m+m_1} B_{ij}^{q_0} T_{ij}^{q_0} = \sum_{q_0=1}^{n} B_{ij}^{q_0}, \qquad (10)$$

where m - the number of boundary nodes in the discrete model.

With a large number of tests, you can get a fairly accurate scene. $B_{ij}^{q_0}$ from the ratio

$$B_{ij}^{q_0} = \frac{N_{ij}^{q_0}}{N}, \qquad (11)$$

where $N_{ij}^{q_0}$ - number of stops in a node $(i, j)^q$ during the initial walk from the point $(i, j)^0$ for N tests.

Thus, taking into account (10), (11), we write the solution of the boundary value problem (1), (2), (9) obtained by the Monte Carlo method for the point $(i, j)^0$:

$$P_{ij}^0 = Y_{ij}^0 = \sum_{q=1}^{n} B_{ij}^{q_0} = \frac{1}{N} \sum_{q=1}^{n} N_{ij}^{q_0}. \qquad (12)$$

Since the five-point approximation scheme is adopted, the flow in $(i, j)^0$ - m node can be determined by the formula [7,8]

$$Q_{ij}^0 = \sum_{r=1}^{4} K_{ij}^r (P_{ij}^r - P_{ij}^0),$$

Where P_{ij}^r - potentials in neighboring $(i, j)^0$ - m nodes, which are determined by the Monte Carlo method when wandering from the mentioned nodes according to formula (12); K_{ij}^r - function values $K(x_1, x_2)$ between $(i, j)^0$ - m node and adjacent to it $(i, j)^r$ - m. Taking into account (9) and (12), we have

$$Q_{ij}^{0}=\sum_{r=1}^{4}K_{ij}^{r}(\sum_{q=1}^{n}\frac{N_{ij}^{qr}}{N}-1)=\frac{1}{N}\left[\sum_{r=1}^{4}K_{ij}^{r}N_{ij}^{r}\right]+$$

$$+,.\quad .\quad .,-\sum_{r=1}^{4}K_{ij}^{r}(1-N_{ij})+,.\quad .\quad .,+\left[\sum_{r=1}^{4}K_{ij}^{r}N_{ij}^{nr}\right]=$$

$$=Q_{1\nu}+Q_{2\nu}+Q_{3\nu}+,.\quad .\quad .,-Q_{\nu\nu}+,...,Q_{n\nu}.$$

The terms obtained on the right side are the coefficients of influence and mutual influence along the flow on $(i,j)^{\nu}$ - th singular point, i.e. ν - th column of the matrix A_Q. Starting to wander from nodes adjacent to singular points, we get the entire matrix A_Q.

Let now for the boundary value problem (1), (7), (9) conditions of the second kind on the inner contours

$$q(x_1,x_2)\Big/_{\Gamma_\nu}=1,\qquad \nu=\overline{1,n}. \tag{13}$$

In each test, the value of the random variable ξ when passing through $(i,j)^{r}$ - th internal node increases by φ_{ij}^{r}:

$$\varphi_{ij}^{r}=\begin{cases}0, & if\quad (i,j)^{r}\in\Gamma_{g},\\ \varphi_{ij}^{r}, if & (i,j)^{r}\in\Gamma_{g},\ g=\overline{1,n},\end{cases}$$

and its mathematical expectation is Y_{ij}^{0}:

$$M_{\xi}=Y_{ij}^{0}=\sum_{g=1}^{n}C_{ij}^{g_0}\varphi_{ij}^{g_0},$$

where $C_{ij}^{g_0}$ - the probability that a wandering particle will pass through an internal node $(i,j)^{g}$ when starting a wander from a point $(i,j)^{0}$.

Taking into account (13), when the particle passes through a special node of the second kind, we obtain [6,7]

$$\varphi_{ij}^{g_0}=-\frac{1}{\sum_{r=1}^{4}K_{ij}^{r}}.$$

With a large number of tests, a fairly accurate estimate is given by the expression

$$C_{ij}^{g_0}\approx\frac{N_{ij}^{g_0}}{N},$$

where $N_{ij}^{g_0}$ - number of passes through a special node of the second kind $(i, j)^g$ when wandering out of a node $(i, j)^0$ the coefficients of influence of which must be determined; N - the total number of passes through all nodes of the second kind when walking from a node $(i, j)^0$.

The solution of the boundary value problem (1), (2), (13) obtained by the Monte Carlo method for the point $(i, j)^0$, will be

$$P_{ij}^0 = \frac{1}{N}\sum_{g=1}^{n} N_{ij}^{g_0}\left(-\frac{1}{\sum_{r=1}^{4} K_{ij}^r}\right) = -\frac{1}{N}\sum_{g=1}^{n} N_{ij}^{g_0}\left(-\sum_{r=1}^{4} K_{ij}^r\right)^{-1}. \quad (14)$$

Then $(i, j)^\nu$ - special node of the second kind, then the right side of expression (14) contains the sum of elements) ν - th row of the matrix A_P (8).

Let us now turn to the consideration of the problem of optimizing extractions from an oil reservoir, taking into account geological and field restrictions, based on the use of a modification of the simplex algorithm for solving the linear programming problem.

Under constraints of some special form, which will be discussed below, we will determine the maximum value of the linear form (4) with constraints (5), (6), in which $A = A_P$.

Let's introduce n - dimensional residual vector

$$q^* \geq 0.$$

Let us carry out some formal transformations of expression (5), (6):

$$A_q + Eq^* = B,$$

$$E_q + A^{-1}q^* = A^{-1}B,$$

allowing for the existence A^{-1}:

$$C_q + CA^{-1}q^* = CA^{-1}B.$$

Because $Z_0 = CA^{-1}B = const,$ then

$$(-CA^{-1}q^*) \to \max \Rightarrow (c, q) \to \max.$$

Denote

$$A^{-1}B = B^*,$$

$$-CA^{-1} = C^*,$$

$$A^{-1} = A^*.$$

Then (4) – (6) reduces to the problem

$(c,q) \to \max$ (15)

at $A^* q^* \leq B,$ (16)

$q^* \geq 0.$ (17)

If a $B^* \geq 0$ и A^* - symmetric Adamard matrix [9], then the solution of problem (15) - (17) (called in [10] the problem of the required form), obtained by the simplex method, has a number of features:

1. Each column of the matrix of coefficients of expression of non-basic vectors through basis vectors can only be a guide once; the guide element is always on the main diagonal.:

2. If the column vector of the matrix A^* problem (15) - (17) has a negative estimate C_j^* at least at one iteration of the solution, then it will definitely enter the optimal basis.

3. The optimal basis for a linear programming problem of the required form does not depend on the magnitude of the components of the vector of right-hand sides of the constraints.

Recalculation of estimates of out-of-basic vectors C_j^* when introduced into the basis e - vector is carried out according to the formula [11]

$$\overset{*}{C}{}_j^{k} = \overset{*}{C}{}_j^{k-1} - \frac{\overset{*}{C}{}_e^{k-1} \overset{*}{a}{}_{ej}^{k-1}}{\overset{*}{a}{}_{ee}^{k-1}}, \qquad (18)$$

where $\overset{*}{C}{}_j^{k}$ - recalculated on k - th iteration score; $\overset{*}{C}{}_j^{k-1}$ - the same assessment for $(k-1)$ - th iteration; $\overset{*}{C}{}_e^{k-1}$ - estimate of the vector introduced into the basis on $(k-1)$ - th iteration; $\overset{*}{a}{}_{ej}^{k-1}$ - the guide element of the vector introduced into the basis on $(k-1)$ - th iteration; $\overset{*}{a}{}_{ee}^{k-1}$ - matrix element A^*, standing at the intersection of the lads guide and the column whose score is being recalculated.

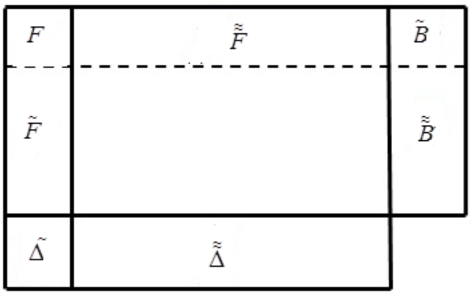
Property 2 allows us to use the idea of Kuhn-Tucker, described in [11], about the block introduction of all vectors with negative estimates into the basis, if on k - th iteration, introduce into the basis all the vectors introduced on $(k-1)$ - th iteration. To do this, we transfer all the vectors introduced into the basis to the left (rearranging the rows at the same time so that the elements of the main diagonal are again on it (Pic. 1).

Row vector $(\tilde{\Delta}, \tilde{\tilde{\Delta}})$ obtained by permutation in the string C^*. The column vector is obtained similarly $(\tilde{B}, \tilde{\tilde{B}})$, at $F, \tilde{F}, \tilde{\tilde{F}}$ - parts of the rearranged matrix A^*;

$\tilde{\Delta}^k = \tilde{\tilde{\Delta}}\ \tilde{\tilde{\Delta}} - F^{-1} \tilde{\tilde{F}}.$

We do not distinguish between a column vector and a row vector, so there is no transposition sign. Because F - symmetric matrix, then the vector q can be found by solving a system of linear equations

$F_q = \tilde{\Delta}.$



Pic.1. Scheme of rearrangement of elements of the main diagonal: $\vec{\Delta}$ - the vector of estimates entered into the basis of the matrix columns A^*; $\tilde{\tilde{\Delta}}$ - the vector of estimates of the remaining columns of the matrix A^*.

To solve this system, the program uses the square root method [12]. If on k - th iteration $\tilde{\tilde{\Delta}}^k \geq 0,$ then the optimal basis is found. The optimal solution is obtained by a formula similar to (18):

$$\tilde{\tilde{\Delta}}_{onm} = \tilde{\tilde{B}} - \tilde{F}F^{-1}\tilde{B}.$$

Designated $F^{-1}\vec{B} = Y$ and find the value Y from system solution

$$FY = \vec{B}.$$

Decision vector q_{opt} has the form

$$q_{opt} = (0, \tilde{\tilde{B}}_{opt}).$$

The objective function after optimization can be calculated by the formula

$$Z = Z_0 + \Delta Z = cq_{opt},$$

where $\Delta Z = -qY.$

The algorithm considered above is used to solve the problem of optimizing extractions from an oil reservoir, taking into account geological and technical limitations. In our notation $A = A_Q, A^* = A_p, B$- vector of maximum allowable depressions; B^* - the vector of flow rates corresponding to the limiting drawdowns; C - vector of oil content coefficients; q- debit vector; q^*- depression residual vector; Z_0 - clean oil production before optimization; Z - clean oil production after shutdown of wells, the numbers of which are included in the optimal basis [13,14].

The program provides for the input of a vector B, or B^*; matrix inversion A not provided. The solution starts with the original plan $q = B^*, q^* = 0.$

As an example, a problem was solved with a matrix of geological and technical constraints $n \times n$ = 7×7 with vectors

$$C = \|0,89; 0,79; 0,06; 0,39; 0,06; 0,05; 0,909\|,$$

$$B = \|1,07; 0,449; 0,409; 1,00; 0,469; 0,248; 0,465\|.$$

The matrix of coefficients of influence and mutual influence has the form:

$$A=\begin{Vmatrix} 0,0241 & -0,000298 & -0,000231 & -0,000833 & -0,000387 & -0,00008 & -0,000391 \\ -0,000297 & 0,0220 & -0,00008 & -0,000410 & -0,00308 & -0,00008 & -0,000418 \\ -0,000217 & 0,00006 & 0,01205 & -0,000351 & -0,0098 & -0,000381 & -0,000211 \\ -0,000834 & -0,000469 & -0,000358 & 0,01241 & 0,000910 & -0,000380 & -0,000090 \\ -0,000410 & -0,00328 & -0,0001 & -0,000896 & 0,042 & -0,000103 & -0,000110 \\ -0,000126 & -0,00006 & -0,000360 & -0,000382 & -0,000102 & 0,01072 & -0,000364 \\ -0,000236 & -0,000410 & -0,000156 & -0,000090 & -0,000105 & -0,000361 & 0,01650 \end{Vmatrix}$$

Calculation accuracy ε is equal to 0,00001. The result of solving the problem is the vector of optimal flow rates $q_{opt} = 1,081;0,508;0;1,042;0;0;0,468$. Initial value of the linear form $Z_0 = 2,701$ increases by $\Delta Z = 0,037$ $Z_{opt} = Z_0 + \Delta Z = 2,738$.

The proposed mathematical model and calculation algorithm using the method of statistical tests for solving applied problems of the distribution of selections is useful in the introduction of digital technologies in the development of oil fields.

Literature

1. Samarsky A.A., Mikhailov A.P. Mathematical modeling: ideas, methods, examples: 2nd ed., corrected. - M.: Fizmatlit, 2002. - 320 p.
2. Vdovin V.V. Optimization of closed linear stochastic systems with linear feedback. Statistical methods of control theory //Abstracts of the IV All-Union Conference. Frunze, 1978. - M.: Science. 1978. - S.65-67.
3. Ермаков С.М., Михайлов Г.А. Курс статистического моделирования. - М.: Наука. - 1976. - 319 с.
4. Kirichenko A.G., Botov N.G. Construction of economical algorithms for simulating random variables in the method of aggregate functions//Interuniversity technical scientific collection. - Tagonrog: TRTI. - 1975 - Issue 1. - P.162-165.
5. Sirazetdinov T.K., Klestov E.A. On the Problem of Optimal Speed in Distributed Systems with Random Properties // Automation and Telemechanics. - 1972. - No. 3. - P.36-42.
6. Sadullaev R., Suvonov O.O. Application of the method of statistical tests for solving problems of analysis of distributed control systems. Abstracts All-Union. school seminar "Actual problems of statistical modeling and its applications", -T: 1989
7. Sadullaev R., Suvonov O.O. Mathematical modeling of a hydrodynamic control system with distributed parameters. All-Union. Meeting on management issues. (Abstracts) - T: 1989
8. Suvonov O.O., Bazarov M.B. Some issues of optimal control of a system with distributed parameters (on the example of oil production). Computing technologies", V.6, № 12. Publishing House of the Siberian Branch of the Russian Academy of Sciences, Novosibirsk -2006. 4 pages
9. Bellman R. Introduction to the theory of matrices.- M.: Nauka. - 1969. - 367 p.
10. Meerov M.V., Litvak B.L. Static optimization of multi-connected systems of large dimensions // Avtomatika i telemekhanika.-1970. - Number 3. - P.143-154.Гасс С. Линейное программирование. - М. Мир. 1961. - 303 с.

11. Galkin L.N. Solution of diffusion problems by the Monte Carlo method. M.: Science. - 1976. - 95 p.
12. Suvonov O.O. Numerical algorithm of computational experiment of the applied optimal control problem in systems with distributed parameters. "Bulletin of TUIT: Management and Communication Technologies" 2021, 2 (46) 1 UDK 62-50:276.681. 10 pages.
13. Suvonov O.O. Mathematical model of control of a hydrodynamic object with distributed parameters (on the example of oil fields). Central Asian journal of mathematical theory and computer sciences http://cajmtcs.centralasianstudies.org/index.php/CAJMTCS Volume: 03 Issue: 04 | Apr 2022 ISSN: 2660-5309. 5 pages.