
On The Accuracy of the Finite Element Method on the Example of Problems about Natural Oscillations

Nasirov Ismail Azizovich
Fergana Polytechnic Institute

Annotation: The article discusses the accuracy of the finite element method on the example of problems of natural vibrations. To assess the accuracy of the developed program, a three-dimensional problem of natural oscillations of a hollow elastic cylinder of finite length with a fixed left and free right end is solved. The principle of possible displacements and the kinematic boundary condition are involved in the mathematical problem of natural vibrations of a cylinder. The obtained results are analyzed in order to detect mechanical effects.

Keywords: principle of possible displacements, axisymmetric, torsional and bending vibrations, finite element method, natural frequencies for torsional vibrations.

Introduction

The aim of this study is to evaluate the convergence of the finite element method on the example of various test problems and compare the obtained solutions with experimental data.

For this purpose, an elastic rod of length l with a cross section in the form of a round ring was considered. The inner and outer radius of the ring are r_1 and r_2 , respectively. The left end of the rod is fixed, while the right end is free (Fig. 1).

The eigenfrequencies of the bending, longitudinal and torsional eigenfrequencies of the rod are sought.

It is known that for a long rod the elementary solution of the problem, obtained in the framework of the hypothesis of flat sections [3], is the following:

a) longitudinal vibrations:

the equation of motion has the form

$$\frac{d^2u}{dx^2} - \frac{1}{c_0^2} \frac{d^2u}{dt^2} = 0, \quad (1)$$

border conditions

$$x = 0: u = 0; x = l: EF \frac{du}{dx} = 0, \quad (2)$$

Natural frequencies are determined by the formula

$$\omega = \frac{\pi(2k-1)c_0}{2l}, \quad (k = 1, 2, \dots), \quad (3)$$

$$\text{where: } c_0 = \sqrt{\frac{E}{\rho}}$$

b) torsional vibrations: equations of motion

$$\frac{\partial^2 \theta}{\partial x^2} - \frac{1}{C_k^2} \frac{\partial^2 \theta}{\partial t^2} = 0,$$

(4)

border conditions

$$x = 0: \theta = 0; x = l: GI_k \frac{\partial \theta}{\partial x} = 0 \quad (5)$$

Natural frequencies are determined by the formula

$$\omega = \frac{\pi(2k-1)C_k}{2l}, \quad (k=1,2,\dots) \quad (6)$$

where: $C_k = \sqrt{\frac{GI_k}{\rho I_0}}$

for circular bar $I_r = I_0$

c) bending vibrations: the equation of motion

$$EI \frac{\partial^4 W}{\partial x^4} - \rho F \frac{\partial^2 W}{\partial t^2} = 0, \quad (7)$$

border conditions

$$x = 0: \frac{\partial W}{\partial x} = 0, W = 0; x = l: \frac{\partial}{\partial x} \left(EI \frac{\partial^2 W}{\partial t^2} \right) = 0, EI \frac{\partial^2 W}{\partial t^2} = 0. \quad (8)$$

The natural frequency is determined by the formula $\omega_n = \frac{\mathcal{H}_k}{l^2} \sqrt{\frac{EI}{\rho F}}$,

where: \mathcal{H}_k – root of the equation $ch \mathcal{H} \cos \mathcal{H} = -1$ and is equal to

$$\mathcal{H}_k = \frac{\pi}{2}(2k - 1), \quad (k=1,2,\dots) \quad (9)$$

Statement and solution of the problem

To assess the accuracy of the developed program, a three-dimensional problem of natural vibrations of a hollow elastic cylinder of finite length with a fixed left and free right end is solved (Fig. 2).

The principle of possible displacements (10),

$$\delta A = - \int_0^{2\pi} \int_0^l \int_{r_1}^{r_2} (\sigma_{ij} \delta \varepsilon_{ij} + \rho \ddot{u} \delta \vec{u}) r \, dr dz d\varphi = 0 \quad (10)$$

and kinematic boundary condition.

$$Z = 0: u_i = 0, i = r, z, \varphi \quad (11)$$

Here r, z, φ are cylindrical coordinates.

The solution of the problem is sought in the form:

$$\vec{u} = \{u_r, u_\varphi, u_z\} \quad (12)$$

$$\varepsilon_{rz} = \frac{1}{2} \left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z} \right), \quad \varepsilon_{\varphi z} = \frac{1}{2} \left(\frac{\partial u_\varphi}{\partial z} + \frac{1}{r} \frac{\partial u_z}{\partial \varphi} \right) \quad (13)$$

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2 \mu \varepsilon_{ij} \quad (14)$$

As you know, when $n=0$ (n is the number of harmonics), the problem is divided into two independent ones: the problem of symmetric oscillations ($u_r = u_r^*(r, z, \varphi)$, $u_z = u_z^*(r, z, \varphi)$, $u_\varphi = 0$) and the problem of rotational oscillations ($u_r = u_z = 0$, $u_\varphi = u_\varphi^*(r, z, \varphi)$). For $n=1$, the problem of bending vibrations arises.

Within the framework of the one-dimensional theory of the resistance of materials based on the hypothesis of flat sections [3], the analogues of the above three problems on the longitudinal, torsional and transverse vibrations of a circular beam. For $n \geq 2$, the three-dimensional problem has no one-dimensional analogues.

The mathematical formulations of the three indicated variants of the three-dimensional problem (axisymmetric, torsional and bending) are obtained from (10,12,13,14).

The posed three-dimensional problem was solved by the finite element method. The area occupied by the body was divided into triangles by ring elements (Fig. 3).

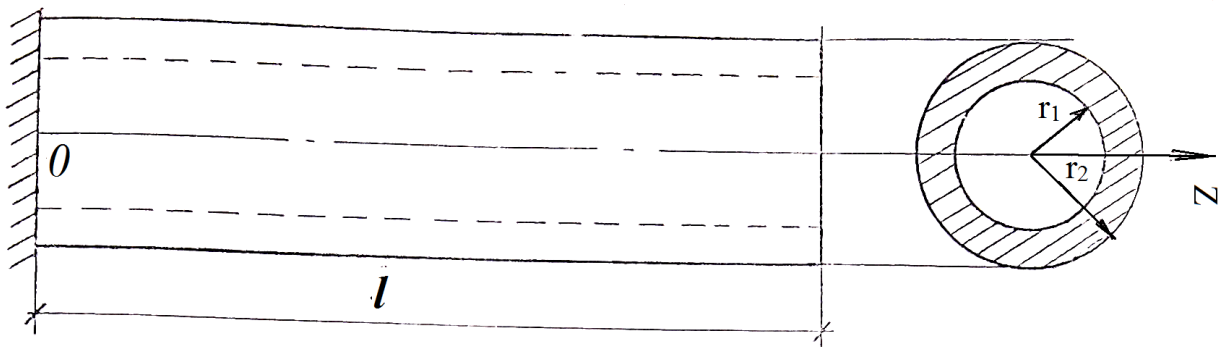


Fig. 1

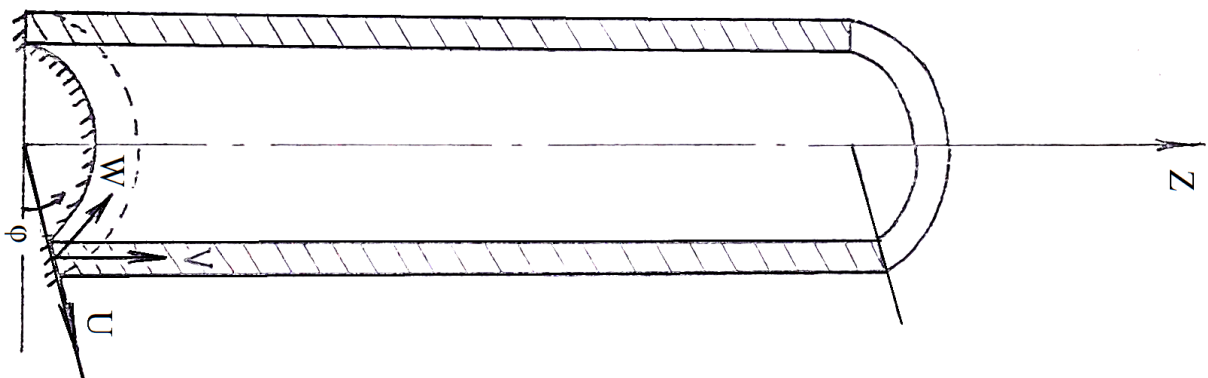


Fig. 2

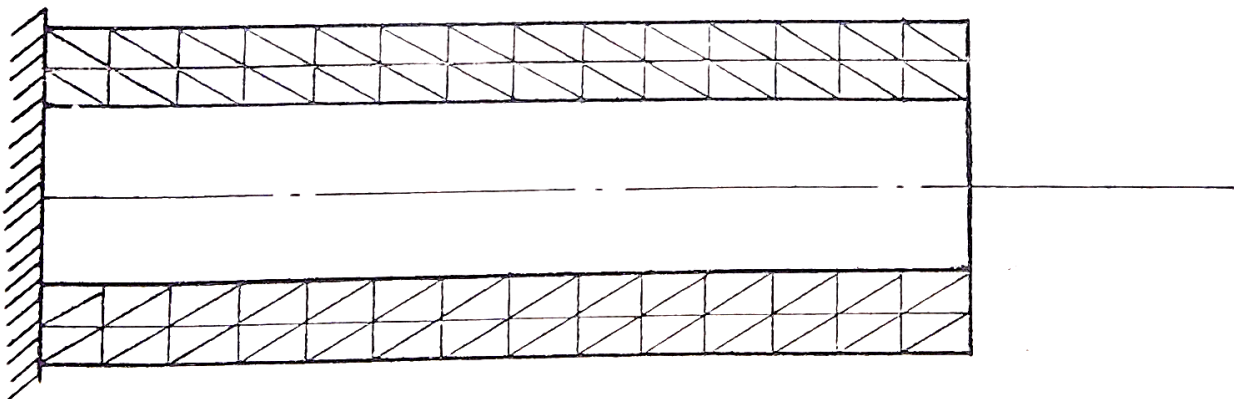


Fig. 3

Results conclusions and conclusions

The results of natural frequencies of the cylinder were compared with the natural frequencies of the rod (1-9), calculated using the hypothesis of flat sections [3]. The results of comparing the frequencies obtained for different values of the ratio of the length of the cylinder l to its outer radius r_2 are shown in tables 1-3.

Natural frequencies for torsional vibrations, determined by the hypothesis of flat sections and by the theory of elasticity, are the same regardless of the ratio l/r_2 (table 3).

Comparison of the calculation results in the case of axisymmetric oscillations (Table 1) showed that the first natural frequencies coincide quite well for all values of l/r_2 , and starting from the second frequency, for $l/r_2 > 10$, there is a significant difference in frequencies.

In the case of bending vibrations (table 2), when $l/r_2 > 20$, there is a good agreement between the results and the exact solution of the rod problem. For $l/r_2 < 20$, the first bending eigenfrequencies differ by $\sim 10\%$, while the fourth and fifth frequencies differ by a factor of 10–15 from the increase in the exact solution. These results were obtained with the following characteristics of the construction material: $E=1.0$; $\nu=0.27$; $\rho=1.0$.

To confirm the reliability of the results obtained (the developed algorithms and programs), using the above algorithm, the problem of axisymmetric natural vibrations of cylinders with rigidly clamped ends and free side surfaces was solved:

a) thin-walled cylinder ($a/b = 0,98$; $L/b = 0,5$) $\nu = 0,3$; $E = 1,0$; $\rho = 1,0$. Table 1

No. natural frequencies	Axisymmetric natural frequencies									
	$l/r_2=2,0$		$l/r_2=10,0$		$l/r_2=20,0$		$l/r_2=100,0$		$l/r_2=200,0$	
	A.H.S	E.T.	A.H.S	E.T.	A.H.S	E.T.	A.H.S	E.T.	A.H.S	E.T.
ω_1	0,052 360	0,052 939	0,010 472	0,010 512	0,005 239	0,005 246	0,001 047	0,001 041	0,000 523	0,000 523
ω_2	0,157 086	0,081 565	0,031 417	0,031 768	0,015 708	0,015 721	0,003 141	0,003 142	0,001 571	0,001 571
ω_3	0,261 829	0,090 855	0,052 365	0,051 474	0,026 182	0,026 133	0,005 236	0,005 234	0,002 618	0,002 618
ω_4	0,366 601	0,126 904	0,073 320	0,068 465	0,036 660	0,036 416	0,007 331	0,007 332	0,003 665	0,003 665
ω_5	0,471 413	0,165 859	0,094 282	0,077 358	0,047 141	0,046 892	0,009 426	0,009 387	0,004 712	0,004 710

Table 2

No. natural frequencies	Axisymmetric natural frequencies									
	$l/r_2=2,0$		$l/r_2=10,0$		$l/r_2=20,0$		$l/r_2=100,0$		$l/r_2=200,0$	
	A.H.S	E.T.	A.H.S	E.T.	A.H.S	E.T.	A.H.S	E.T.	A.H.S	E.T.
ω_1	0,035 216	0,020 654	0,001 408	0,001 367	0,000 352	0,000 352	0,000 014	0,000 014	0,0000 035	0,0000 035
ω_2	0,220 684	0,056 288	0,008 827	0,007 036	0,002 206	0,002 071	0,000 088	0,000 088	0,0000 220	0,0000 220
ω_3	0,617 920	0,078 650	0,024 716	0,016 286	0,006 179	0,005 346	0,000 247	0,000 247	0,0000 617	0,0000 617
ω_4	0,210 887	0,112 021	0,048 435	0,026 439	0,012 108	0,009 514	0,000 484	0,000 483	0,0001 210	0,0001 210
ω_5	2,001 674	0,119 440	0,080 067	0,047 189	0,020 016	0,017 124	0,000 800	0,000 800	0,0002 001	0,0002 001

Table 3

No. natural frequencies	Torsional natural frequencies									
	$l/r_2=2,0$		$l/r_2=10,0$		$l/r_2=20,0$		$l/r_2=100,0$		$l/r_2=200,0$	
	A.H.S	E.T.	A.H.S	E.T.	A.H.S	E.T.	A.H.S	E.T.	A.H.S	E.T.
ω_1	0,032 853	0,032 941	0,006 570	0,006 611	0,003 285	0,003 290	0,000 657	0,000 653	0,000 328	0,000 313
ω_2	0,098 560	0,099 212	0,019 712	0,019 837	0,009 856	0,009 918	0,001 971	0,001 982	0,000 985	0,000 996
ω_3	0,164 268	0,166 519	0,032 853	0,033 077	0,016 426	0,016 532	0,003 285	0,003 305	0,001 64	0,001 666
ω_4	0,229 975	0,235 49	0,045 994	0,046 337	0,022 997	0,023 149	0,004 599	0,004 638	0,002 299	0,002 334
ω_5	0,295 682	0,308 458	0,059 136	0,059 128	0,029 568	0,029 769	0,005 913	0,005 908	0,002 956	0,003 002

Note: A.H.S.– according to the hypothesis of flat sections

E.T. – elasticity theory.

The result obtained was compared with the results of work [8], obtained in the framework of the Kirchhoff-Love hypothesis (Table No. 4).

Table 4

natural frequencies	ω_1	ω_2	ω_3	ω_4	ω_5
Obtained in [8] on the theory of shells	1,027	1,383	2,371	3,951	6,056
Obtained using the developed algorithm	1,009	1,312	2,312	3,901	6,001

b) for a cylinder

b) for a cylinder ($a/b = 0,4$; $L/b = 1,0$) $\nu = 0,45$; $\mu = 1,0$; $\rho = 1,0$

The result obtained was compared with the results of [9]

Table 5

natural frequencies		ω_1^2	ω_2^2	ω_3^2	ω_4^2	ω_5^2
Obtained in [9]		16,801	31,054	43,019	74,281	79,226
Obtained using the developed algorithm	at 120 elements	18,113	34,446	48,178	83,611	90,781
	at 320 elements	17,379	32,306	45,001	77,982	83,583
	at 750 elements	16,998	31,475	43,634	75,451	80,504

Here: ν - Poisson's ratio, E - Young's modulus, μ - shear modulus, ρ - material density, a - inner radius, b - outer radius, L - cylinder length.

To test the developed algorithm and programs, the problem of bending natural vibrations of a particular structure was considered - a reinforced concrete ventilation pipe of the Armenian NPP 150 m high.

The obtained periods of bending eigenoscillations were compared with the experimental data [3.2]. The comparison results are shown in table 6.

Table 6

Name of the study	Periods of natural oscillations				
	T_1	T_2	T_3	T_4	T_5
Experiment [8]	1,63	0,51	0,20	-	-
Algorithm developed by the author	1,598	0,487	0,211	0,131	0,089

Literature

1. Mirsaidov M.M., Troyanovsky E.I. Dynamics of inhomogeneous systems with allowance for internal dissipation and wave entrainment of energy. Tashkent: Fan, 1990.108s.
2. Vibration in technology. Handbook, Mechanical Engineering, 1978, 351 p.
3. Babakov I.V. Theory of Oscillations.- M.: I968, 420 p.
4. Bate K., Wilson E. Numerical methods of analysis and FEM. M.: Stroyizdat, 1982.448s.
5. Muller D.E. A Method for Solving Algebraic Equations Using an Automatic Computer. MathematicalTabl., Oktober, 1956.

6. Amosov A.A., Dubinsky Yu.A., Kopchenova N.V. Computational methods. St. Petersburg. 2014, . "Doe". 672 p.
7. Mirsaidov M., Nosirov A.A., Mayboroda V.P., Troyanovsky I.E. Three-dimensional problem of natural oscillations of a rectilinear rod with an annular section. On Sat. "Questions in Mechanics", vol. 17, Tashkent, publishing house "Fan" UzSSR, 1982.
8. Mirsaidov M. Steady oscillations of axisymmetric viscoelastic shells. Abstract of the dissertation for the degree of candidate of physical and mathematical sciences. M., 1976.
9. Matvienko V.P. Optimization, deformation and dynamic calculation of a viscoelastic axisymmetric body with mixed conditions at the boundary. Abstract of the dissertation for the degree of candidate of physical and mathematical sciences. M., 1977.
10. Mirsaidov M.M., Khudainazarov Sh.O. Spatial natural vibrations of viscoelastic axisymmetric structures. Magazine of Civil Engineering. 2020. 96(4). Pp. 118–128. DOI: 10.18720/MCE.96.10
11. Nosirov A.A., I.A. Nasirov I.A. Natural and Forced Vibrations of Axisymmetric Structure Taking into Account the Viscoelastic Properties of the Base. Middle European Scientific Bulletin, VOLUME 18 Nov 2021
12. M M Mirsaidov¹, A A Nosirov² and I A Nasirov² Modeling of spatial natural oscillations of axisymmetric systems. Journal of Physics: Conference Series 1921 (2021) 012098 IOP Publishing doi:10.1088/1742-6596/1921/1/012098.
13. R. Abdikarimov¹, D. Usarov², S. Khamidov³, O. Koraboshev⁴, I. Nasirov⁵ and A. Nosirov⁵ Free oscillations of three-layered plates. IOP Publishing doi:10.1088/1757-899X/883/1/012058.
14. Mirziyod Mirsaidov^{1*}, Abdurasul Nosirov², and Ismoil Nasirov² Spatial forced oscillations of axisymmetric inhomogeneous systems. E3S Web of Conferences 164, 02009 (2020).
15. D.A. Sagdullayeva¹, Sh.A. Maxmudova¹, F.F. Adilov¹, R.A. Abirov¹, I.O. Khazratkulov² and I.A. Nasirov³ On stability of slopes in mountain zones. Case study. IOP Conf. Series: Journal of Physics: Conf. Series 1425 (2020) 012016.
16. Abdukarimov, B. A., & Kuchkarov, A. A. (2022). Research of the Hydraulic Resistance Coefficient of Sunny Air Heaters with Bent Pipes During Turbulent Air Flow. Journal of Siberian Federal University. Engineering & Technologies, 15(1), 14-23.
17. Abdukarimov, B. A. (2021). Improve Performance Efficiency As A Result Of Heat Loss Reduction In Solar Air Heater. International Journal of Progressive Sciences and Technologies, 29(1), 505-511.
18. Malikov, Z. M., & Madaliev, M. E. (2020). Numerical simulation of two-phase flow in a centrifugal separator. Fluid Dynamics, 55(8), 1012-1028.
19. Маликов, З. М., & Мадалиев, М. Э. (2021). Численное моделирование течения в плоском внезапно расширяющемся канале на основе новой двухжидкостной модели турбулентности. Вестник Московского государственного технического университета им. НЭ Баумана. Серия «Естественные науки», (4 (97)), 24-39.
20. Madraximov, M. M., Abdulkhaev, Z. E., & ugli Inomjonov, I. I. (2022). Factors Influencing Changes In The Groundwater Level In Fergana. International Journal of Progressive Sciences and Technologies, 30(2), 523-526.
21. Arifjanov, A., Otaxonov, M., & Abdulkhaev, Z. (2021). Model of groundwater level control using horizontal drainage. Irrigation and Melioration, 2021(4), 21-26.

22. Худайкулов, С. И., & Муминов, О. А. У. (2022). МОДЕЛИРОВАНИЯ МАКСИМАЛЬНОЙ СКОРОСТИ ПОТОКА ВЫЗЫВАЮЩЕЙ КАВИТАЦИЮ И РЕЗКОЙ ПЕРЕСТРОЙКИ ПОТОКА. *Universum: технические науки*, (2-2 (95)), 59-64.
23. АБДУЛҲАЕВ, З., & МАДРАХИМОВ, М. (2020). Гидротурбиналар ва Насосларда Кавитация Ҳодисаси, Оқибатлари ва Уларни Бартараф Этиш Усуллари. *Ўзбекгидроэнергетика” илмий-техник журнали*, 4(8), 19-20.
24. ugli Mo‘minov, O. A., Maqsudov, R. I., & qizi Abdukhalilova, S. B. (2021). Analysis of Convective Fins to Increase the Efficiency of Radiators used in Heating Systems. *Middle European Scientific Bulletin*, 18, 84-89.
25. Усмонова, Н. А., Негматуллоев, З. Т., Нишонов, Ф. Х., & Усмонов, А. А. (2019). Модели закрученных потоков в строительстве Каркидонского водохранилища. *Достижения науки и образования*, (12 (53)), 5-9.
26. Абдукаримов, Б. А., Аббасов, Ё. С., & Усмонова, Н. У. (2019). Исследование рабочих параметров солнечных воздухонагревателей способы повышения их эффективности. *Матрица научного познания*, (2), 37-42.
27. Мадрахимов, М. М., & Абдулхаев, З. Э. (2019). Насос агрегатини ишга туширишда босимли сув узатгичлардаги ўтиш жараёнларини ҳисоблаш усуллари. *Фарғона Политехника Институту Илмий–Техника Журнали*, 23(3), 56-60.
28. Mamadalievich, M. M., & Erkinjonovich, A. Z. Principles of Operation and Account of Hydraulic Taran. *JournalNX*, 1-4.
29. Сатторов, А. Х. (2016). СУЩЕСТВОВАНИЕ И ПРЕДСТАВЛЕНИЕ ОГРАНИЧЕННОГО РЕШЕНИЯ ОДНОГО КВАЗИЛИНЕЙНОГОДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ. In *Вузовская наука-региону* (pp. 126-132).
30. Мадхадимов, М. М., Абдулхаев, З. Э., & Сатторов, А. Х. (2018). Регулирования работы центробежных насосов с изменением частота вращения. *Актуальные научные исследования в современном мире*, (12-1), 83-88.
31. Abdikarimov, R., Usarov, D., Khamidov, S., Koraboshev, O., Nasirov, I., & Nosirov, A. (2020, July). Free oscillations of three-layered plates. In *IOP Conference Series: Materials Science and Engineering* (Vol. 883, No. 1, p. 012058). IOP Publishing.
32. Nosirov, A. A., & Nasirov, I. A. (2021). Natural and Forced Vibrations of Axisymmetric Structure Taking into Account the Viscoelastic Properties of the Base. *Middle European Scientific Bulletin*, 18, 303-311.
33. qizi Abdukhalilova, S. B. (2021). Simplified Calculation of the Number of Bimetallic Radiator Sections. *CENTRAL ASIAN JOURNAL OF THEORETICAL & APPLIED SCIENCES*, 2(12), 232-237.
34. Maqsudov, R. I., & qizi Abdukhalilova, S. B. (2021). Improving Support for the Process of the Thermal Convection Process by Installing. *Middle European Scientific Bulletin*, 18, 56-59.
35. Мадрахимов, М. М., Абдулхаев, З. Э., & Ташпулатов, Н. Э. (2019). Фарғона Шаҳар Ер Ости Сизот Сувлари Сатҳини Пасайтириш. *Фарғона Политехника Институту Илмий–Техника Журнали*, 23(1), 54-58.
36. Hamdamov, M., Mirzoyev, A., Buriev, E., & Tashpulatov, N. (2021). Simulation of non-isothermal free turbulent gas jets in the process of energy exchange. In *E3S Web of Conferences* (Vol. 264, p. 01017). EDP Sciences.

37. Рашидов, Ю. К., Орзиматов, Ж. Т., & Исмоилов, М. М. (2019). Воздушные солнечные коллекторы: перспективы применения в условиях Узбекистана. In Экологическая, промышленная и энергетическая безопасность-2019 (pp. 1388-1390)
38. Рашидов, Ю. К., Исмоилов, М. М., Орзиматов, Ж. Т., Рашидов, К. Ю., & Каршиев, Ш. Ш. (2019). Повышение эффективности плоских солнечных коллекторов в системах теплоснабжения путём оптимизации их режимных параметров. In Экологическая, промышленная и энергетическая безопасность-2019 (pp. 1366-1371).
39. Madraximov, M. M., Abdulkhaev, Z. E., & Orzimatov, J. T. (2021). GIDRAVLIK TARAN QURILMASINING GIDRAVLIK HISOBI. Scientific progress, 2(7), 377-383.
40. Rashidov, Y. K., & Orzimatov, J. T. (2022). SOLAR AIR HEATER WITH BREATHABLE MATRIX ABSORBER MADE OF METAL WIRE TANGLE. Scientific-technical journal, 5(1), 7-13.
41. Усаров, М. К., & Маматисаев, Г. И. (2019). КОЛЕБАНИЯ КОРОБЧАТОЙ КОНСТРУКЦИИ КРУПНОПАНЕЛЬНЫХ ЗДАНИЙ ПРИ ДИНАМИЧЕСКИХ ВОЗДЕЙСТВИЯХ. In Научный форум: технические и физико-математические науки (pp. 53-62).
42. Abdugarimov, B., O'tbosarov, S., & Abdurazakov, A. (2021). Investigation of the use of new solar air heaters for drying agricultural products. In E3S Web of Conferences (Vol. 264, p. 01031). EDP Sciences.
43. Усаров, М. К. & Маматисаев, Г. И. (2014). К динамическому расчету коробчатой конструкции здания. ME' MORCHILIK va QURILISH MUAMMOLARI, 86.
44. Bekzod, A. (2020). Relevance of use of solar energy and optimization of operating parameters of new solar heaters for effective use of solar energy. IJAR, 6(6), 16-20.
45. Madraximov, M. M., Nurmuxammad, X., & Abdulkhaev, Z. E. (2021, November). Hydraulic Calculation Of Jet Pump Performance Improvement. In International Conference On Multidisciplinary Research And Innovative Technologies (Vol. 2, pp. 20-24).
46. Hamdamalievich, S. A., & Nurmuxammad, H. (2021). Analysis of Heat Transfer of Solar Water Collectors. Middle European Scientific Bulletin, 18, 60-65.
47. Madaliev, M. E. U., Maksudov, R. I., Mullaev, I. I., Abdullaev, B. K., & Haidarov, A. R. (2021). Investigation of the Influence of the Computational Grid for Turbulent Flow. Middle European Scientific Bulletin, 18, 111-118.
48. Madraximov, M., Yunusaliev, E., Abdulhayev, Z., & Akramov, A. (2021). Suyuqlik va gaz mexanikasi fanidan masalalar to'plami. GlobeEdit.
49. Абдукаримов, Б. А. Акрамов, А. А. У., & Абдухалилова, Ш. Б. К. (2019). Исследование повышения коэффициента полезного действия солнечных воздухонагревателей. Достижения науки и образования, (2 (43)).
50. Умурзакова, М. А. Усмонов, М. А., & Рахимов, М. Н. (2021). АНАЛОГИЯ РЕЙНОЛЬДСА ПРИ ТЕЧЕНИЯХ В ДИФФУЗОРНО-КОНФУЗОРНЫХ КАНАЛАХ. Экономика и социум, (3-2), 479-486.
51. Аббасов, Ё. С. & Умурзакова, М. А. (2020). РАСЧЕТ ЭФФЕКТИВНОСТИ ПЛОСКИХ СОЛНЕЧНЫХ ВОЗДУХОНАГРЕВАТЕЛЕЙ. In Современные проблемы теплофизики и энергетики (pp. 7-8)