Simulation of Spatial Own of Vibrations of Axisymmetric Structures

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Annotation: The article deals with natural vibrations of the spatial axisymmetric dome shaped structure. The variational formulation of the problem arising from the principle of possible displacements is carried out. To solve the problem, an algorithm and computer programs have been developed using the semi-analytical finite element method (FEM). The non-axisymmetric natural frequencies and vibration modes of the investigated object have been determined. The results obtained are analyzed in order to detect mechanical effects.

Keywords: non-axisymmetric natural vibrations, spatial axisymmetric structure, spatial vibrations, finite element method, close and multiple natural frequencies.

Introduction and problem statement

The axisymmetric dome-shaped structure shown in Fig. 1 is considered. 1. The lower part of the structure is motionless, its inner and outer surfaces are free, there are no mass forces. Non-axisymmetric natural frequencies and vibration modes of an elastic spatial structure are to be determined.

For the formulation of the problem, a variational equation is used based on the variational formulation of the problem arising from the principle of possible displacements [1]:

$$\delta A = -\int_0^{2\pi}\int_0^{l}\int_{r_1}^{r_2}\left(\sigma_{ij}\delta\varepsilon_{ij} + \rho\ddot{u}_i\delta u_i\right)r\ drdzd\varphi = 0,\ (1)$$

$$i, j = r, z, \varphi$$

The kinematic boundary condition is also used: $z = 0:\quad u_i = 0, i = r, z, \varphi$ (2)

Here r, z, φ are cylindrical coordinates, ui, Ϭij , εij are the components of the displacement vector, stress and strain tensors, respectively, ρ is the density of the material, δ ui, δ εij are variations of displacements and strains.

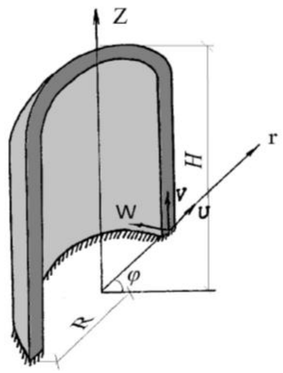

Figure 1. Domed structure.

The solution of the problem

The strain tensor and displacement vector components are related by the Cauchy relations[1]:

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r}, \quad \varepsilon_{\varphi\varphi} = \frac{1}{r}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u}{r}, \varepsilon_{zz} = \frac{\partial u_z}{\partial z},$$

$$\varepsilon_{r\varphi} = \frac{1}{2}(\frac{1}{r}\frac{\partial u_r}{\partial \varphi} + \frac{\partial u_\varphi}{\partial r} - \frac{u_\varphi}{r}),$$

$$\varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right), \varepsilon_{\varphi r} = \frac{1}{2}\left(\frac{\partial u_\varphi}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \varphi}\right),$$

The components of the stress and strain tensors are related by the relations of the generalized Hooke's law

$\sigma_{ij} = \lambda \varepsilon_{kk}\delta_{ij} + 2\,\mu\varepsilon_{ij}$ (4)

where λ, μ are the Lame parameters, δi j is the Kronecker symbol.

Non-axisymmetric eigenoscillations of problem (1)-(4) are sought, that is, motions having the form

$$u_r = u_r^*(r,z)\cos n\varphi \cdot \cos \omega t$$

$$u_\varphi = u_\varphi^*(r,z)\sin n\varphi \cdot \cos \omega t (6)$$

$$u_z = u_z^*(r,z)\cos n\varphi \cdot \cos \omega t$$

where uz* are the desired functions, n is the number of harmonics (n=0,1,2,…),ω is the natural frequency.

For n=0, the problem splits into two independent ones, the problem of axisymmetric vibrations: ur= ur(r,z,t), uz = uz(r, z,t), u_φ= 0 and the problem of rotational vibrations:

ur≡uz= 0, u_φ= u_φ(r,z,t).

For n=1, the problem of bending vibrations arises. In the framework of the one-dimensional theory of the strength of materials, the analogues of the above three problems are the problems of longitudinal, torsional, and transverse vibrations of a circular beam [2].

For n≥ 2, the three-dimensional problem has no one-dimensional analogues.

The problem posed is solved in this work by the semi-analytical (FEM) finite element method [3]. The area occupied by the body (structure) is divided into annular elements of a triangular section with a linear approximation of the displacement inside each element (Fig. 2).

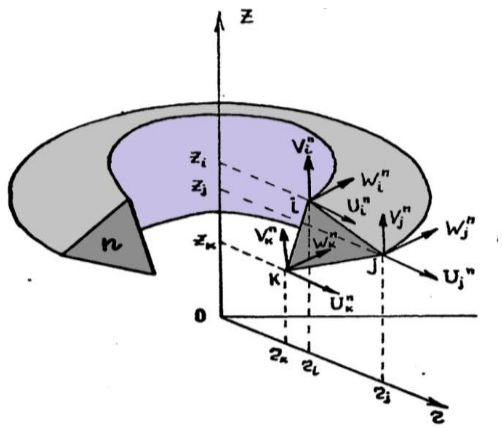

Figure 2. Ring elements of a triangular section with a linear approximation of the displacement inside each element

The procedure of the finite element method reduces the problem of natural oscillations to the algebraic problem of finding non-zero solutions of a homogeneous system

$[K] - \omega^2[M]\{x\}$(7)

where [K] is the stiffness matrix, [M] is the mass matrix, {x} is the vector of nodal displacements.

The order of system (7) depends on the number of finite elements into which the domain is divided. In the present work, the maximum order of system (7) reached 800.

The stiffness matrix has a band structure, the structure of the mass matrix depends on whether the mass is considered to be distributed over the element or concentrated at the grid nodes. In the first case, the mass matrix has a band structure, in the second, it has a diagonal structure.

In this paper, the solution of the algebraic problem (7) i.e. the search for the roots of the characteristic determinant of the system was carried out using the Muller method [4].

After determining the eigenfrequencies, the problem of finding eigenforms is reduced to constructing a nontrivial solution to an algebraic system with zero determinant and zero right hand side.

The solution of the system was found by the square root method [5], that is, the rank of the matrix was checked; then the corresponding row and column are excluded, and the solution of the remaining system of inhomogeneous linear algebraic equations is found.

To implement the above algorithm, a Fortran program was compiled; to test its performance and convergence of the method, a number of numerical experiments were carried out [6].

Results and discussions

Table 1 shows the first five natural frequencies of the structure for each harmonic (n=1,2,3…). For these values of eigenfrequencies, the corresponding eigenmodes of vibrations are plotted in axonometry (Fig. 3(a,b,c,d,e)). The results shown in Table 1 were obtained with the following structural characteristics E = 2.0; ν = 0.27; ρ = 0.00667.

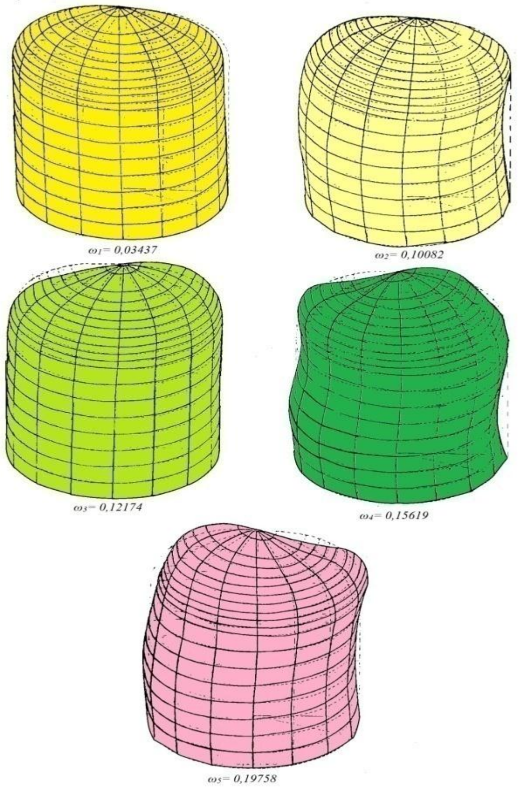

Figure 3(a). Non-axisymmetric waveforms for n=1

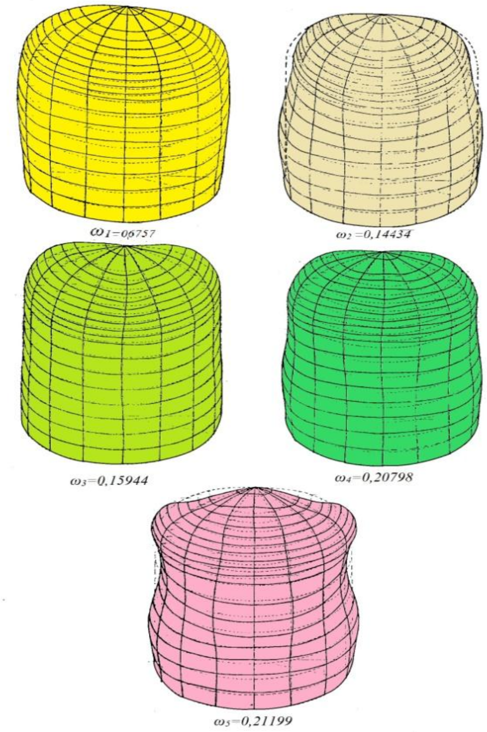

Figure 3(b). Non-axisymmetric waveforms for n=2

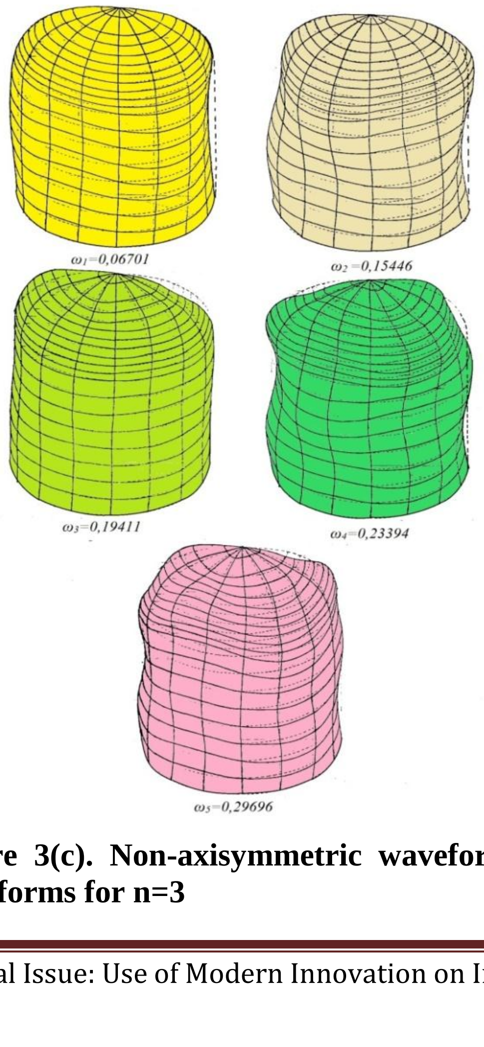

Figure 3(c). Non-axisymmetric waveforms for n=3

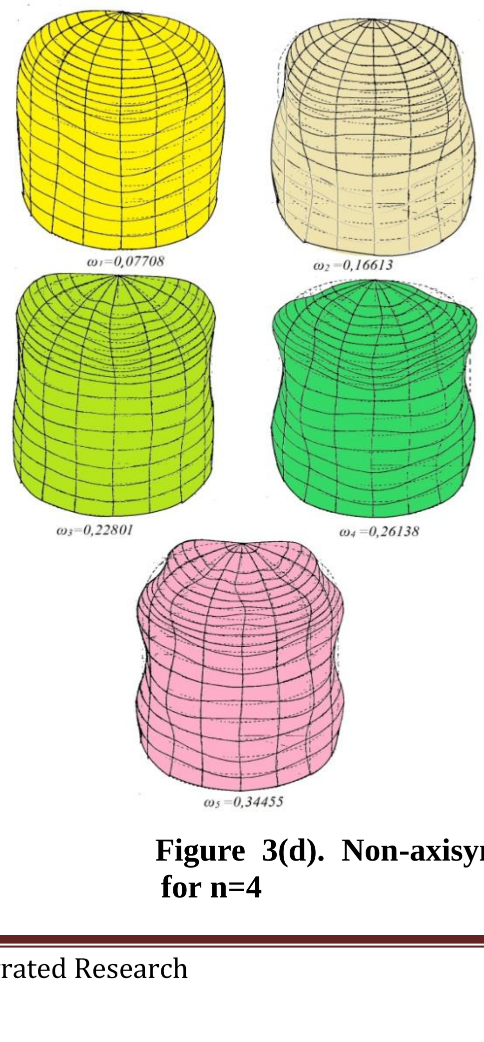

Figure 3(d). Non-axisymmetric waveforms for n=4

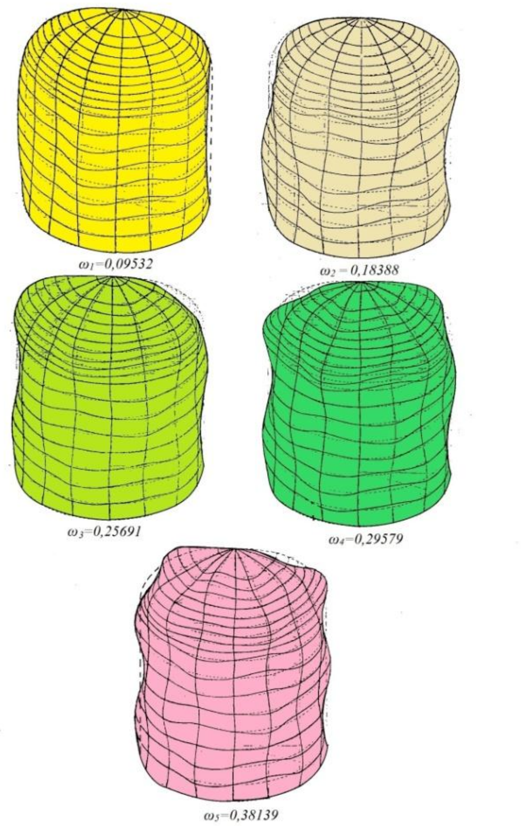

Figure 3(e). Non-axisymmetric waveforms for n=5

The analysis of natural frequencies of the structure at different values of the ratio H /R (H is the height of the structure, R is its outer radius) within H⁄(R=) 1.5÷2.5 showed that the frequencies and forms change places. In this case, the greater the ratio H / R (within the specified limits), the greater the frequency of torsional vibration to the lowest, i.e. to the bending frequency. In the case when the ratio H / R is closer to the minimum value (within the given limits), then the torsional vibration frequency corresponds to the third or fourth natural frequency of the structure.

Findings and Conclusions

Figure 4 shows the characteristic dependences of the first three natural frequencies (flexural, axisymmetric and torsional vibrations) on the relative height of the structure - H / R. All three frequencies increase with increasing relative height. The most significant is the difference in the rate of increase for different frequencies, as a result of which there are points of intersection of the graphs. At these points, the oscillation frequencies become multiple. A similar, only more complicated situation also arises for higher natural frequencies, it may turn out that in the vicinity of one specific parameter value, three curves intersect, related to frequencies corresponding to different numbers and harmonics.

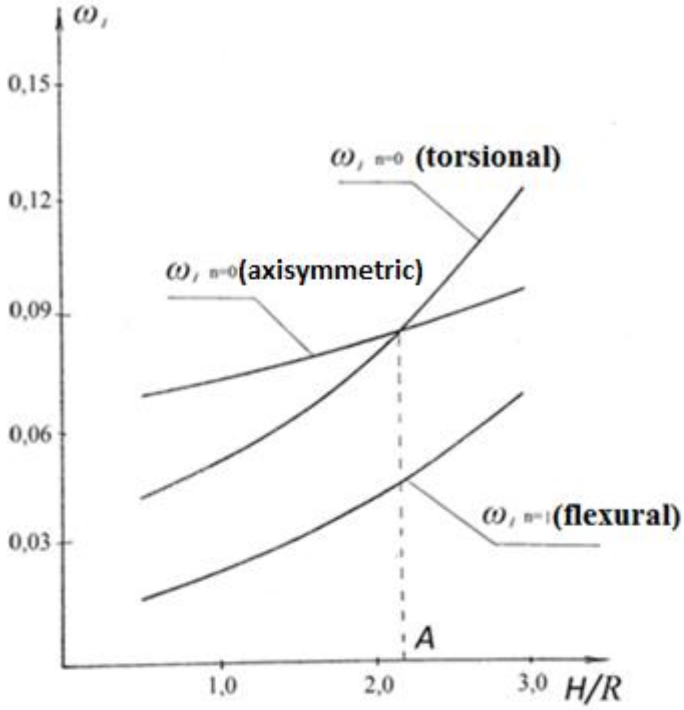

Figure 4. Dependence of natural frequencies on the relative height of the structure H / R.

Further, natural frequencies were analyzed in order to find multiples or close ones. This analysis was carried out for various ratios of the geometric parameters of the structure [7].

As can be seen from Table 1, the second axisymmetric frequency is close (multiple) to the third frequency at n=1: ($\omega 2n=0$ = 0,121517 $\approx \omega 3n=1$= 0,121740).

Spatial natural frequencies of the structure Table 1.

Number of harmonics	natural frequencies				
	ω_1	ω_2	ω_3	ω_4	ω_5
n= 0 (axisymmetric)	0,080	0,122	0,162	0,177	0,233
n = 0 (torsional)	0,066	0,195	0,309	0,412	0,523
n =1	0,034	0,101	0,122	0,156	0,198
n = 2	0,068	0,144	0,159	0,208	0,212
n = 3	0,067	0,154	0,194	0,234	0,297
n = 4	0,077	0,166	0,228	0,261	0,345
n = 5	0,095	0,188	0,257	0,296	0,381

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