The Advantage of Solving Dynamic Problems in the Mathcad Package

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Annotation: This article shows the convenience and advantage of calculating the dynamics issues using MathCAD package, which is one of the sections of theoretical mechanics. Calculating issues using mathematical packages takes less time and is easier to understand by viewing their graphs.

Keywords: MathCAD package, differential equations, theoretical mechanics, dynamic problems, Koshi problem.

Initially, we will consider some problems of theoretical mechanics, which are reduced to solving ordinary differential equations (ODES). To solve them, the built-in functions of the MathCAD package will be used.

MathCAD function for solving an ordinary differential equation of the m -th order. Recall that an ordinary differential equation of the m-th order is called the equation

$$y^{(m)} = f\left(t, y, y', \dots, y^{(m-1)}\right), \qquad (1)$$

where $y^{(m)} = \frac{d^m y(t)}{dt^{(n)}}$ –means an m-th order derivative with respect to the variable t.

The Koshi problem for such an equation is to find such a function y(t), which is a solution to equation (1) and satisfies the initial conditions:

$$y(t_0) = y_0, y'(t_o) = y_1, \dots, y^{(m-1)}(t_0) = y_{m-1}. \qquad (2)$$

To solve the Koshi problem considered in the MathCAD package, the odesolve function is most often used, which is referred to in the Given decision block. In this block, the m-th order differential equation itself and the initial conditions are set.

The appeal to the odesolve function has the form:

$$\text{odesolve}(t, t_k, n), \qquad (3)$$

where *t* - is the name of the variable (the argument of the desired solution);

t_k — is the value specifying the end of the integration interval (the beginning of the interval t_0); it is specified in the initial conditions set in the Given block by the following construction: $y = (t_0) = y_0, y'(t_0) = y_1$ and etc. (bold characters are used in this case <<equals>>);

n - is an optional parameter that determines the number of intervals into which the segment (t_0, t_k) is divided. If this parameter is omitted, then the calculation of *n* occurs inside the odesolve function.

The result of the work is the value of the solution of the differential equation at time t.

The use of the odesolve function is shown in the following example.

Example 1. The problem of free oscillations of a dynamical system. Let the load Q rest on an elastic spring. The deviation of the load from the equilibrium position and at the initial moment of time is denoted by y_0, and the velocity values are y_1 (initial conditions). The deviation down is positive, up is negative. In the equilibrium position, the weight is balanced by the spring elasticity. It is assumed that the force tending to return the load to the equilibrium position – the so-called restoring force - is proportional to the deviation, i.e. equal to $- k \cdot y$, where k is some constant value for this spring (<<spring stiffness>>). It is proposed that the movement of the load Q is hindered by a resistance force directed in the direction opposite to the direction of movement and proportional to the speed of movement of the load, i.e. the force $- \lambda \cdot v = -\lambda \cdot \frac{dy}{dt}$, where $\lambda = const \geq 0$ (shock absorber). Find the function $y(t)$ −the coordinate of the movement of the load.

Decision. The differential equation of the movement of the load on the spring according to Newton 's II law has the form:

$$Q \cdot \frac{d^2 y}{dt^2} = -k \cdot y - \lambda \cdot \frac{dy}{dt}. \qquad (4)$$

where $k, \lambda \geq 0$.

The solution of the Koshi problem is shown in the MathCAD document.

$Q := 20 \qquad\qquad \lambda := 40$

$k := 100$

$y_0 := 0.01 \qquad y_1 := 0$

Given

$Q \cdot y''(t) = -k \cdot y(t) - \lambda \cdot y'(t)$

$y(0) = y_0 \qquad y'(0) = 0$

$y_c := odesolve(t, 6, 100) \qquad\qquad t := 0.0, 0.1 .. 6$

$y_a(t) := e^{-t} \cdot \left[0.01 \cdot cos(2 \cdot t) + \left(\frac{0.01}{2} \cdot sin(2 \cdot t) \right) \right]$
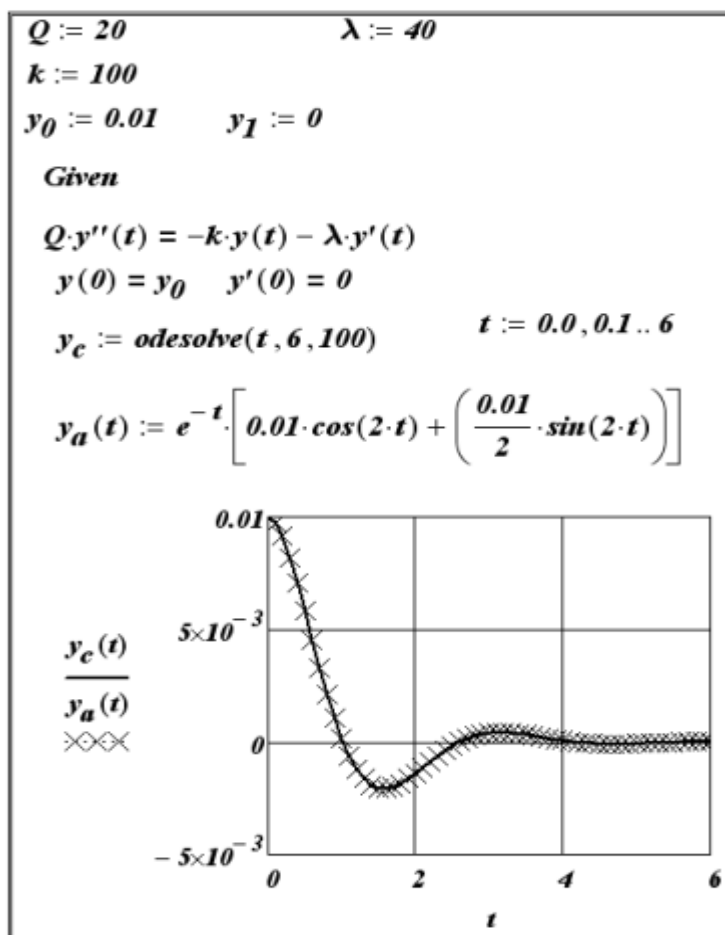
Fig.1. Solution of the Koshi problem for example 1.

All system parameters are set here. To enter a stroke (the designation of the derivative), press [Ctrl]+[F7]. The paper presents graphs of the numerical solution (curve $y_c(t)$ −solid line) and the analytical solution (curve $y_a(t)$ −cretics). A good coincidence of the two curves is

visible.

To illustrate the possibilities of MathCAD, consider an example with a spring, the rigidity of which varies depending on the sign of the deviation of the load.

Example 2. The problem of free oscillations of a dynamic system with variable stiffness. In the conditions of example 1. we make the following assumption, namely, the stiffness of the spring is set as follows:

$$k(y) = \begin{cases} 100, & y(t) \geq 0 \\ 20, & y(t) < 0. \end{cases} \qquad (5)$$

Decision. To set the variable stiffness (5), we will use the if function. In Fig.2. A document solving the Cauchy problem is shown. Comparing the graphs of numerical solutions $y_c(t)$ (Fig.1.), one can notice the difference between the solutions obtained. The analytical solution cannot be obtained, and this graph of the curve $y_a(t)$ (Fig. 2.) is missing.
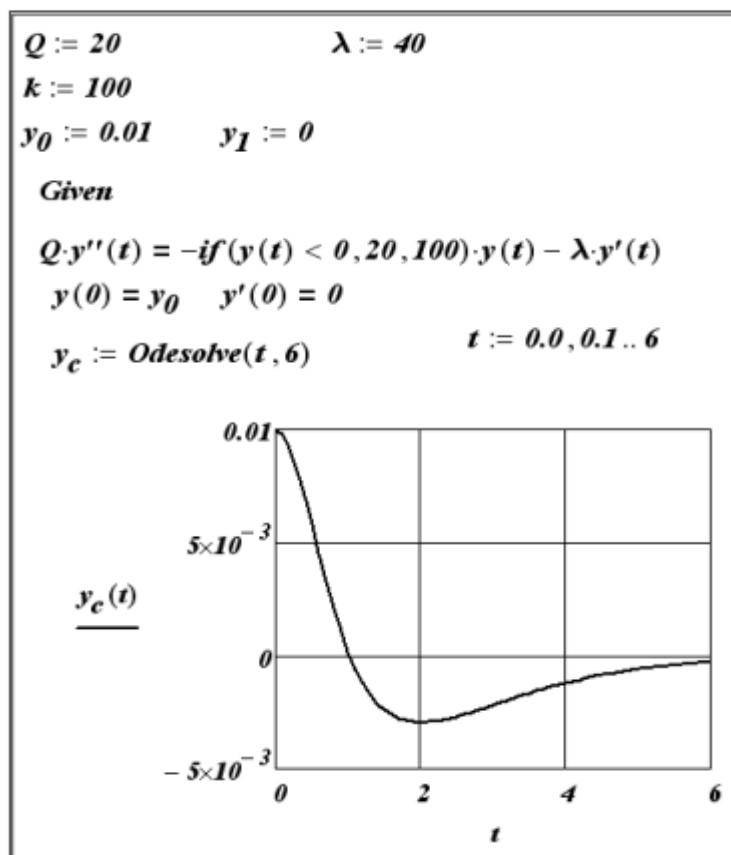

$Q := 20 \qquad \lambda := 40$

$k := 100$

$y_0 := 0.01 \qquad y_1 := 0$

Given

$Q \cdot y''(t) = -if(y(t) < 0, 20, 100) \cdot y(t) - \lambda \cdot y'(t)$

$y(0) = y_0 \qquad y'(0) = 0$

$t := 0.0, 0.1 .. 6$

$y_c := Odesolve(t, 6)$

Fig.2. Solution of the Cauchy problem for example 2.

MathCAD functions for solving systems of first-order differential equations. Many mexanical processes they are described by systems of equations containing derivatives of several unknown functions for the same independent variable. Consider such a system of equations:

$$\begin{cases} y_1' = f_1(t, y_1, y_2, \ldots, y_m), \\ y_2' = f_2(t, y_1, y_2, \ldots, y_m), \\ \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\ y_m' = f_m(t, y_1, y_2, \ldots, y_m). \end{cases} \qquad (6)$$

where $y_1, y_2, \ldots, y_m$ −are the desired functions of the variable t;

$f_1, f_2, \ldots, f_m$ −set functions of several variables

We will consider the Koshi problem for the system (6):

$$y_1(t_0) = y_1, y_2(t_0) = y_2, \ldots, y_m(t_0) = y_m. \tag{7}$$

where $y_1, y_2, \ldots, y_m$ −set values.

In this case, the solution is calculated on the interval (t_0, t_k).

Note that the first-order differential equation is a special case of the system (6) for m = 1.

The Koshi problem for an ODE system is reduced to solving the Koshi problem for an ODE of the form (1). By introducing new variables:

$$y_1' = y' = y_2, y_2' = y'' = y_3, \ldots, y_m' = y^{(m)}, \tag{8}$$

we come to the first - order ODE system:

$$\begin{cases} y_1' = y_2, \\ y_2' = y_3, \\ \ldots \ldots \ldots \ldots \\ y_{m-1}' = y_m \\ y_m' = f(t, y_1, y_2, \ldots, y_m) \end{cases} \tag{9}$$

with initial conditions (2):

To solve first–order ODE systems, the following functions are defined in the package:

➢ rkfixed – the Runge-Kutta method with a fixed step;

➢ Rkadapt – Runge – Kutta method with adaptive step selection;

➢ Bulstoer – the Bulirsch–Stern method.

The reference to these functions has the form:

$rkfixed(y, t_0, t_k, n, D); Rkadapt(y, t_0, t_k, n, D); Bulstoer(y, t_0, t_k, n, D).$

The arguments of the functions are the same:

y - is the vector of initial conditions of dimension m, the projections of which are determined by the values (7);

t_0, t_k-are the boundary points of the interval on which the solution of the differential equation is sought;

n- is the number of points (not counting the initial one) in which the values of the desired solution are calculated;

D- is a vector function of two variables *D(x,y)* containing a vector of the first derivatives, i.e. the right part of the systems (6) or (8).

The result of these functions is a matrix of size $(n + 1) \times (n + 1)$ the first column of which is the values of the argument *t* in which the solution was calculated. The second column and the subsequent ones are the numerical values of the solutions $y_1(t), y_2(t), \ldots, y_m(t)$ at points $t_0, t_1, \ldots, t_n$ (the first column of the matrix).

Example 3. Using the *rkfixed* function to solve the problem Koshi of example 1, reducing it to the Koshi problem for a system of first-order ODES.

Decision. Let's introduce new variables:

$y_1(t) = y(t) -$ the desired solution of the ODE;

$y_2(t) = y'(t) -$is the first derivative of the solution of the ODE.

Then we get a system of first - order ODES:

$$\begin{cases} y_1' = y_2, \\ y_2' = -\dfrac{k}{Q}y_1 - \dfrac{\lambda}{Q}y_2. \end{cases} \qquad (11)$$

The initial conditions are $y_1(t_0) = 0,01, y_1(t_0) = 0$.

The solution of this system is given in the document shown in Figure 3. Note that the ORIGIN variable is set to 1 (to match the indexes of variables with the system designations (11). The variables y_1, y_2 are elements of the vector, and therefore their index expressions are entered after pressing the "left square bracket" key. The variables y_1, y_2 (set the initial conditions) are scalar quantities,

and their indices are entered after pressing the key $[\cdot]$ – decimal point.
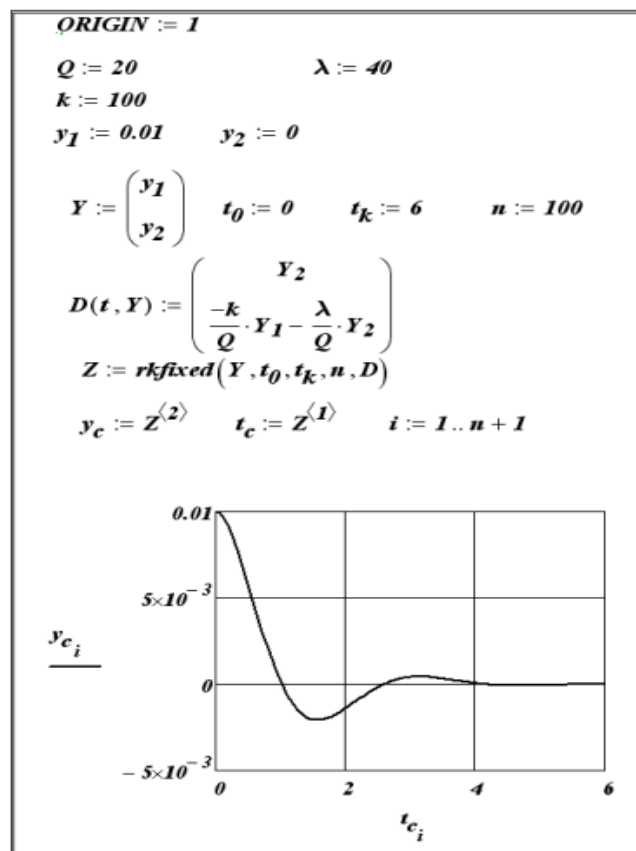

Fig. 3. Solution of the Koshi problem for a system of ordinary differential equations (example 3)

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